Write your full name in the answer sheet and indicate your course number.

**Instructions for MA 452 students:** Do only four of the five problems for full credit. Be sure to indicate which four are to be graded. Each question is worth 25 points.

**Instructions for MA 502 students:** Do all five problems for full credit. Each question is worth 20 points.

1. (i) Use the intermediate value theorem to show that $2^x = 3x$ has at-least one real solution in $(0, 1)$.
   (ii) Use Mean Value Theorem to show that $(x - 1)/x < \ln(x) < x - 1$ for $x > 1$ and hence $\ln(1 + x) < x$ for $x > 0$.

2. Show that if $x > 0$, then
   \[ 1 + \frac{1}{2}x - \frac{1}{8}x^2 \leq \sqrt{1 + x} \leq 1 + \frac{1}{2}x. \]

3. Let $g(x) = |x^3|$ for $x \in \mathbb{R}$. Find $g'(x)$ and $g''(x)$ for $x \in \mathbb{R}$, and $g'''(x)$ for $x \neq 0$. Show that $g'''(0)$ does not exist.

4. (i) Let $f(x) = x^2$ for $0 \leq x \leq 1$. For the partition $P_n = \{0, 1/n, 2/n, \ldots, (n - 1)/n, 1\}$, calculate $L(P_n, f)$ and $U(P_n, f)$, and show that $\int_0^1 f(x) \, dx = \int_0^1 f(x) \, dx = \frac{1}{3}$.
   (Use the formula $1^2 + 2^2 + 3^2 + \ldots + m^2 = \frac{1}{6}m(m + 1)(2m + 1)$.)
   (ii) Express the limit as a Riemann integral and evaluate the limit by evaluating the Riemann integral:
   \[ \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \sin\left(\frac{i\pi}{n}\right) \]

5. Define $f$ by
   \[ f(x) = \begin{cases} 
   x^2, & \text{if } x \in \mathbb{Q} \\
   0, & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. 
   \end{cases} \]
   Show that $f$ is continuous at only one point and is differentiable there.