[1] Consider the finite difference scheme

\[ u_{j}^{n+1} = \frac{2\mu}{1 + 2\mu}(u_{j+1}^{n} + u_{j-1}^{n}) + \frac{1 - 2\mu}{1 + 2\mu}u_{j}^{n-1} \]

(a) Show that the scheme is consistent with the PDE \( u_t = u_{xx} \), where \( \mu = \frac{\Delta t}{(\Delta x)^2} = \text{constant} \).
(b) Determine the order of accuracy in \( \Delta t \) of the scheme.

[2] Consider the scheme

\[ \frac{u_{j}^{n+1} - u_{j}^{n}}{\Delta t} = \frac{1}{2} \frac{u_{j+1}^{n+1} - u_{j-1}^{n+1}}{2\Delta x} + \frac{1}{2} \frac{u_{j+1}^{n} - u_{j-1}^{n}}{2\Delta x} \]

for discretizing \( u_t = u_x \).
(a) Find the truncation error and determine the order of accuracy.
(b) Perform a stability analysis for the scheme via the usual Fourier analysis.

[3]
(a) Consider solving the PDE \( u_t + 2u_x = 0 \) in the domain \( 0 \leq x \leq 1 \). Where would you prescribe the boundary condition so that the problem is well posed? Justify your answer.
(b) Consider now a hyperbolic system

\[ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_t + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

in the domain \( 0 \leq x \leq 1 \). Prescribe a suitable boundary condition so that the problem is well posed.

[4] Determine the accuracy of the (forward-backward) MacCormak scheme

\[ \tilde{v}_m^{n+1} = v_m^n - a\lambda(v_{m+1}^n - v_m^n) \]
\[ v_m^{n+1} = \frac{1}{2}(v_m^n + \tilde{v}_m^{n+1}) - a\lambda(\tilde{v}_{m+1}^{n+1} - \tilde{v}_{m-1}^{n+1}) \]

for solving the PDE \( u_t + au_x = 0 \), where \( \lambda = \Delta t/\Delta x \). This scheme is identical to another well-known scheme for this PDE. Identify that scheme.