[1] (i) Use the three point centered difference formula for the second derivative to approximate \( f''(\pi/2) \), where \( f(x) = \sin(x) \) for \( h = 0.01 \). Find the approximation error.

(ii) Fit the data to the periodic model \( F_3(t) = c_1 + c_2 \cos(2\pi t) + c_3 \sin(2\pi t) \):

\[
(0, 3), (1/2, 1), (1, 3), (3/2, 2)
\]

[2] (i) Consider the equation \( x^3 + x - 2 = 0 \) with root \( r = 1 \). Add the term \( cx \) to both sides and divide by \( c \) to obtain \( g(x) \).

(a) For what \( c \) is Fixed Point Iteration (FPI) locally convergent to \( r = 1 \)?

(b) For what \( c \) will FPI converge fastest?

(ii) Show that the iterative procedure for evaluating the square root of 3 using Newton’s method is: \( x_{n+1} = \frac{x_n + \frac{3}{x_n}}{2} \).

[3]

(i)

\[
\begin{align*}
x_0 &= 0.0 & f[x_0] \\
x_1 &= 0.4 & f[x_1] \\
x_2 &= 0.7 & f[x_2] = 6 \\
f[x_0, x_1] & = 50 \\
f[x_0, x_1, x_2] & = \frac{50}{7} \\
f[x_1, x_2] & = 10
\end{align*}
\]
Determine the missing entries in the table and then determine the quadratic Newton interpolation polynomial.

(ii) Find an upper bound for the error on \([0,2]\) when the degree three Chebyshev interpolating polynomial is used to approximate \(f(x) = \sin(x)\).

(iii) Find the clamped cubic spline that interpolates \((0,1), (1,1), (2,5)\) in the interval \(0 \leq x \leq 2\) with the boundary conditions \(s'_1(0) = 0\) and \(s'_2(2) = 1\).

[4]
(i) Identify for which values of \(x\) there is subtraction of nearly equal numbers, and find an alternate form that avoids the problem for \(f(x) = \frac{1-(1-x)^3}{x}\).

(ii) Convert 9.6 to binary and express it as a floating point number \(fl(x)\) by using the rounding to nearest rule.

(iii) Find the following sum by hand in IEEE double precision computer arithmetic, using the rounding to nearest rule: \((1 + (2^{-51} + 2^{-53})) - 1\).

[5]
(i) Consider the linear system \(x_1 + 2x_2 = 1, 2x_1 + 4.01x_2 = 2\) with approximate solution \(x_c = [3, -1]\). Find the error magnification factor and the condition number (infinity norm) of the associated matrix.

(ii) Given the \(100 \times 100\) matrix \(A\), your computer can solve the 50 problems \(Ax = b_1, \ldots, Ax = b_{50}\) in exactly one minute, using \(A = LU\) factorization methods. How much of the minute was the computer working on the \(A = LU\) factorization.

(iii) Solve the system by finding the \(PA = LU\) factorization and then carrying out the two-step back substitution.

\[
\begin{pmatrix}
-1 & 0 & 1 \\
2 & 1 & 1 \\
-1 & 2 & 0
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix}
= 
\begin{pmatrix}
-2 \\
17 \\
3
\end{pmatrix}
\]