[1] Consider the finite difference scheme

\[
\frac{u_j^{n+1} - u_j^{n-1}}{2\Delta t} = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta x)^2}
\]

for discretizing \( u_t = u_{xx} \).

(i) Determine the order of accuracy. Show that it is consistent.

(ii) Analyze the stability of the scheme.

[2] (i) Derive the Crank-Nicolson scheme for discretizing \( u_t + au_x = 0 \) with second order space-time accuracy.

(ii) Find the amplification factor \( \lambda \). Is it dissipative? Justify your answer.

(iii) Consider the Crank-Nicolson scheme for discretizing \( u_t = u_{xx} \). Find the amplification factor \( \lambda \). Is it dissipative? Justify your answer.

[3] (i) Derive the 5-point Laplace approximation for \( \Delta u = f \). Determine the truncation error.

(ii) Find the approximate solution to \( \Delta u = x + y \) in \([0,1] \times [0,1]\) with \( u = 0 \) on the boundary, using the scheme in (i). You may use \( \Delta x = \Delta y = 1/3 \).

[4] State whether the following statements are true or false:

(i) The upwind scheme \( \frac{u_j^{n+1} - u_j^n}{\Delta t} + a \frac{u_j^n - u_{j-1}^n}{\Delta x} = 0 \) for \( u_t + au_x = 0, a > 0 \), is second order accurate.

(ii) The scheme in (i) for the PDE there is unconditionally stable.

(iii) The scheme in (i) for the PDE there has leading phase error for \( 1/2 < \mu < 1 \) and lagging phase error for \( \mu < 1/2 \).

(iv) The box scheme for \( u_t + au_x = 0 \) is dissipative.

(v) The box scheme for \( u_t + au_x \) is unconditionally stable.

(vi) The box scheme for \( u_t + au_x = 0 \) has phase lead error.

(vii) The Lax-Wendroff for \( u_t + au_x = 0 \) has phase lag error.

(viii) The leap-frog scheme for \( u_t + au_x = 0 \) is conditionally stable.

(ix) The leap-frog scheme is dissipative.

(x) The leap-frog scheme for \( u_t + au_x = 0 \) has phase lag error.

(xi) The forward in time and central in space scheme for \( u_t + au_x = 0 \) is conditionally stable.

(xii) The Lax-Wendroff scheme for \( u_t + au_x = 0 \) is TVD.

(xiii) The Lax-Fredrichs scheme for \( u_t + au_x = 0 \) is TVD.

[5] Consider the explicit scheme

\[
u_j^{n+1} = \frac{1}{2}(u_{j+1}^n + u_{j-1}^n) - \frac{a\mu}{1 + (a\mu)^2}(u_{j+1}^n - u_{j-1}^n)
\]

for solving the PDE \( u_t + au_x = 0 \), where \( \mu = \Delta t/\Delta x \).

(i) Analyze the consistency of this scheme.

(ii) Analyze the Stability of this scheme.