Write your full name in the answer sheet and indicate your course number.

**Instructions for MA 452 students:** Do only 8 of the 10 problems for full credit. Be sure to indicate which 8 are to be graded. Each question is worth 12.5 points.

**Instructions for MA 502 students:** Do all the 10 problems for full credit. Each question is worth 10 points.

[1] Show that the sequence \((c^n/n)\), \(0 < c < 1\), converges and find its limit.

[2] Show that \(f(x) = \cos(x)\) is continuous on \(R\). Hint: You may use the following sum-to-product identity: \(\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\).

[3] Prove or disprove the following:
The function \(f(x) = 1/x\) is uniformly continuous on the interval \((0, \infty)\).

[4] Let \(f(x) = x|x|\). Show that \(f\) is differentiable at \(x=0\) and find its derivative there.

[5] Show that the sequence \((\sin(n\theta\pi))\), where \(\theta\) is a rational number with \(0 < \theta < 1\), diverges.

[6] Show that \(\frac{x}{1+x} \leq \ln(1+x) \leq x\) for \(x > 0\) using the Mean Value Theorem.

[7] Let \(f : (a, b) \to \mathbb{Z}\) be continuous on \((a, b)\) where \(\mathbb{Z}\) is the integers. Show that \(f\) must be a constant function.

[8] Let \(f\) and \(g\) be continuous on \([a, b]\), with \(\int_a^b f(x) dx = \int_a^b g(x) dx\). Show that \(f(c) = g(c)\) for some \(c\) in \([a, b]\).

[9] Evaluate the limit
\[
\lim_{n \to \infty} \frac{e^{1/n} + e^{2/n} + \cdots + e^{(n-1)/n} + e^{n/n}}{n}.
\]

[10] Let
\[
s_n = \frac{\sin(1)}{1^2} + \frac{\sin(2)}{2^2} + \cdots + \frac{\sin(n)}{n^2}
\]
for \(n \in \mathbb{N}\). Show that \((s_n)_{n \in \mathbb{N}}\) is Cauchy.